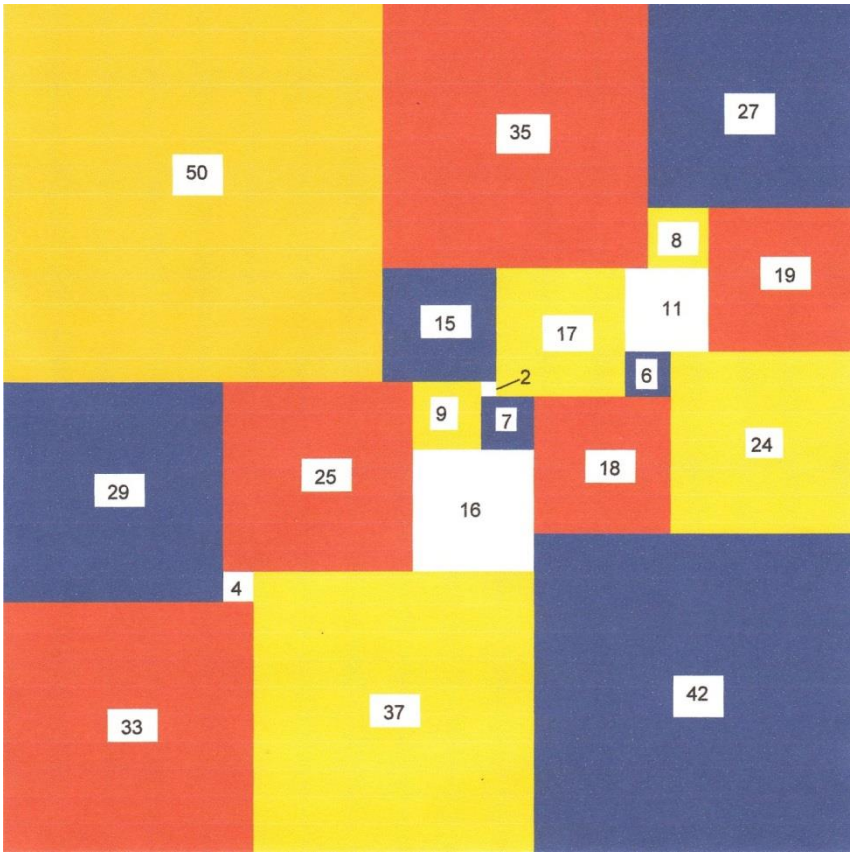
Duijvestijn's perfect squared square

is the answer to the following question: what is the minimum number of different squares needed to cover a larger square?

The squares must have sides of integer length, no two of them may have the same size, and they must cover the larger square without overlapping.



Duijvestijn proved that at least 21 different smaller squares are needed.

Moreover, every tiling of a square by 21 different smaller squares is isomorphic to this Duijvestijn square.

A square (rectangle) with a tiling of *n* smaller pairwise different squares with integer sides is a perfect squared square (rectangle) of order *n*.

The interest in squared squares and rectangles arose around 1900 in recreational mathematics: a simple puzzle, but hard to solve. Non-square perfect squared rectangles were found first in 1925 by Z. Moroń. In 1938 R.P. Sprague published the first perfect squared square of order 55. In 1946 the amateur mathematician T.H. Willcocks constructed a perfect squared square of order 24, which remained the one of the lowest order known until Duijvestijn found his.

Duijvestijn's lowest order perfect squared square was discovered in the night of March 22, 1978 by the DEC-10 mainframe computer of the

University of Twente (Technische Hogeschool Twente at the time), running Duijvestijn's software.

Unlike Willcocks', Duijvestijn's square is simple, it does not contain a smaller perfect squared rectangle.

Duijvestijn started to design his computer programs to search for perfect squared squares and rectangles around 1960, for his Ph.-D research at Eindhoven University of Technology (Technische Hogeschool Eindhoven at the time), inspired by his supervisor prof. dr. C.J. Bouwkamp. But he reached the boundaries of the limited computer capacities of those days. In 1975 the new mainframe computer of the University of Twente, where Duijvestijn had been appointed as full professor in 1965, became operational. About a year later he began to renew and improve his old computer code and to have it running again in intervals, mostly during weekends, on this new computer.

Prof. dr. ir. A.J.W. (Arie) Duijvestijn

The Hague, December 10, 1927
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Enschede, January 21, 1998

Photo of 1989.

At that time (1977-1978) it took around 50 days computing time to reach the result. On a modern notebook it takes 6 seconds (measured in 2018). Processor speed increased enormously since the late 1970's and, even more important for this algorithm, so did the amount of available memory.

Duijvestijn's success was widely appreciated. The Journal of Combinatorial Theory uses his square on its cover. The University of Göttingen erected a monument with his square incised in black granite.

Cat on a home made Duijvestijn square in LEGO®. (by Eric Harshbarger)

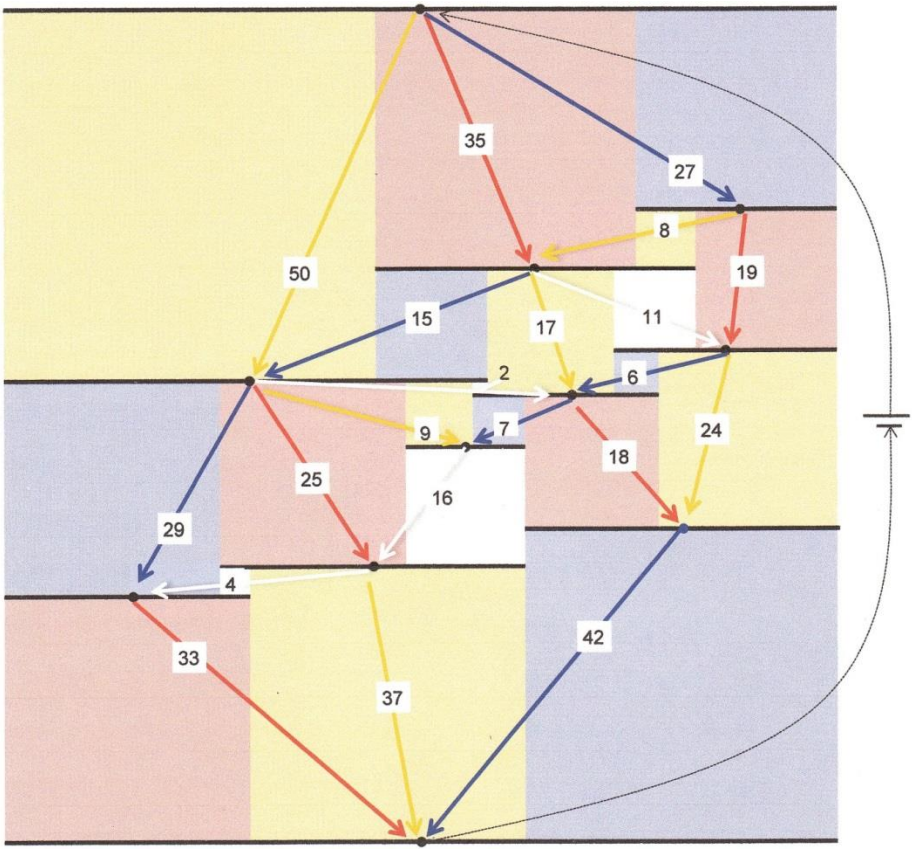
Picture taken from www.squaring.net.

Go there to find everything and more about perfect squared squares and rectangles.

At the heart of Duijvestijn's software was a generator of 3-connected planar graphs.

The graph theory underpinning the generation process was developed by W.T. Tutte.

The mechanism to detect and remove (overwhelming amounts of) automorphic duplicates was invented by Duijvestijn.



The horizontal joints between the square tiles of a squared square can be seen as vertices of a graph. The tiles between the joints are the arcs of the graph.

If we take the arcs to be resistors, all with the same resistance, and we apply the appropriate voltage between top and bottom vertex (top and bottom side of the square), the current through each resistor will be equal to the size of the underlying tile.

The correspondence between squared squares and electrical circuits was discovered and investigated in the late 1930s by four Cambridge students (the 'Trinity Four'): R.L. Brooks, C.A.B. Smith, A.H. Stone, and W.T. Tutte.